Derivation of a key angle for a convex pentagon

The main body of the blog post said that angle A could be deduced from the following equation:

$$\cos\frac{3A}{2} = \frac{\sin\frac{A}{2}}{\sin A}.$$

Express $\frac{3A}{2}$ as $A + \frac{A}{2}$ and apply the formula for $\cos(A+B)$ on the left. On the right, use $\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$. Rearrange:

$$2(\cos A\cos\frac{A}{2} - \sin A\sin\frac{A}{2})\cos\frac{A}{2} = 1$$

$$2\cos^2\frac{A}{2}\cos A - 2\sin A\sin\frac{A}{2}\cos\frac{A}{2} = 1$$

Use the fact that $\cos A = 2\cos^2\frac{A}{2} - 1$ in the first term, and regroup $2\sin\frac{A}{2}\cos\frac{A}{2}$ into $\sin A$ in the second.

$$(1+\cos A)\cos A - \sin^2 A = 1$$

$$\cos A + \cos^2 A - (1-\cos^2 A) - 1 = 0$$

$$2\cos^2 A + \cos A - 2 = 0$$

Now we simply apply the quadratic formula and discard the negative root:

$$\cos A = \frac{-1 \pm \sqrt{1+16}}{4} = \frac{\sqrt{17}-1}{4}$$

$$A \approx 38.6682824925°.$$